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RESEARCH NOTE

STEADY STATE TEMPERATURE DISTRIBUTION IN A DISC WITH RADIAL FLOW OF ELECTRIC CURRENT

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NOMENCLATURE

A, area perpendicular to the direction of current flow;
C, constant of integration;
E, electric potential;
i, electric current;
k, thermal conductivity;
M, quantity defined by equation (7);
N, quantity defined by equation (17);
q, generated heat;
R, electrical resistance;
r, radial coordinate;
T, temperature;
t, thickness of the disc;
ρ, electrical resistivity;
σ, Thomson coefficient.

Subscripts
0, refers to the outside periphery of the disc;
i, refers to the inside periphery of the disc.

IN THE literature, the temperature distribution in cylindrical conductors with electric current flowing in the axial direction has been obtained for various boundary conditions. The results are summarized in the book by Carslaw and Jaeger [1]. In the present study, the steady state temperature distribution in a disc with current flowing in the radial direction is derived for two different boundary conditions; namely, with and without heat transfer at the inner periphery of the disc. The Thomson effect is also considered.

Considering the element of volume of the disc shown in Fig. 1, the energy balance in cylindrical coordinates, for the steady state case with constant properties and heat transfer in the radial direction only, can be written as

$$k\frac{d^2T}{dr^2} + \frac{k}{r}\frac{dT}{dr} + q = 0. \tag{1}$$

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A general solution to equation (1) will be obtained first neglecting the Thomson heat and second considering the Thomson heat. In both cases, two different boundary conditions (zero and finite heat transfer at the inner periphery of the disc) will be used to obtain specific solutions to equation (1).

NEGLECTING THE THOMSON HEAT

Neglecting the Thomson heat, the generated heat will be due to the Joule effect only, thus

$$q = \left(\frac{i^2}{A}\right)\frac{dR}{dr} \tag{2}$$

and according to the Ohm's Law

$$i = \left(\frac{A}{\rho}\right)\frac{dE}{dr} = \frac{dE}{dR} \tag{3}$$

or

$$dR = \left(\frac{\rho}{A}\right)dr. \tag{4}$$

Substituting equation (4) in equation (2) and replacing A by $2\pi rt$, one obtains

$$q = \rho(i/2\pi rt)^2. \tag{5}$$

Substitution of equation (5) in equation (1) and rearrangement yields

$$r^2\frac{d^2T}{dr^2} + r\frac{dT}{dr} + M = 0 \tag{6}$$

where

$$M \equiv (\rho/k)(i/2\pi t)^2. \tag{7}$$

It must be noted that the properties ρ and k are considered to be independent of temperature and location.

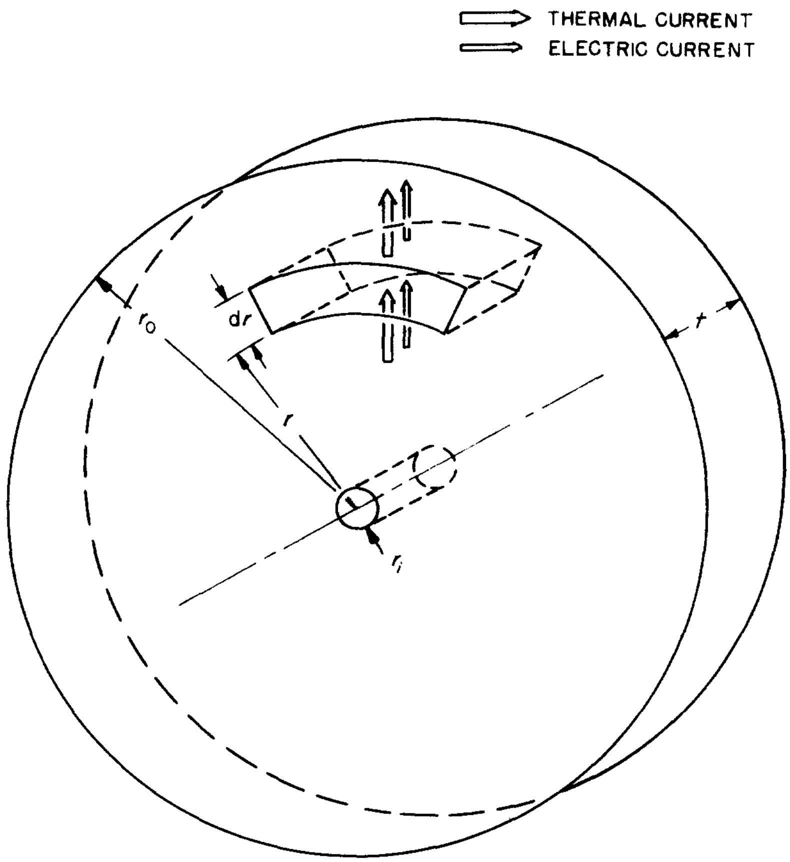

FIG. 1. Element of volume of the disc for heat-transfer analysis.

The general solution of equation (6) is

$$T = -(M/2)(\ln r)^2 + C_1 \ln r + C_2. \quad (8)$$

The specific solutions of equation (8) are obtained for two different boundary conditions.

Case 1

For the boundary conditions

$$\left.\begin{aligned} T &= T_i \quad \text{at} \quad r = r_i \\ T &= T_0 \quad \text{at} \quad r = r_0 \end{aligned}\right\} \quad (9)$$

equation (8) yields

$$T - T_0 = \ln\left(\frac{r_0}{r}\right)\left[\frac{(M/2)\ln(r/r_i) + (T_i - T_0)}{\ln(r_0/r_i)}\right]. \quad (10)$$

Case 2

For the boundary conditions

$$\left.\begin{aligned} \frac{dT}{dr} &= 0 \quad \text{at} \quad r = r_i \\ T &= T_0 \quad \text{at} \quad r = r_0 \end{aligned}\right\} \quad (11)$$

equation (8) yields

$$T - T_0 = M \ln(r_0/r) \ln[\sqrt{(r_0 r)}/r_i]. \quad (12)$$

The reduced temperature distribution in a disc as a function of reduced radial distance for various M values is presented in Fig. 2 for the case $r_i/r_0 = 0{\cdot}2$. The case with no heat transfer at r_i corresponds to $M = 1{\cdot}55$.

The electric potential across the disc (between r_i and r_0) is

$$E_{i0} = \int_{r_i}^{r_0} i \, dR. \quad (13)$$

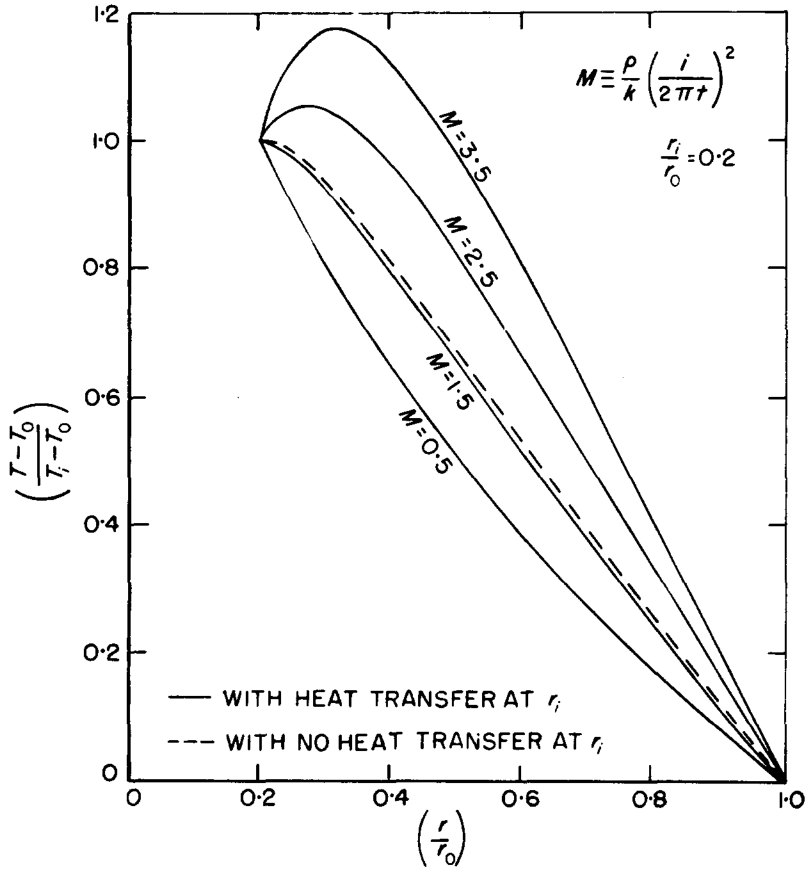

FIG. 2. Temperature distribution in the disc under different heat-transfer conditions.

Substituting equation (4) in equation (13), replacing A by $2\pi rt$ and performing the integration, one obtains

$$E_{i0} = (\rho i/2\pi t) \ln (r_0/r_i). \tag{14}$$

CONSIDERING THE THOMSON HEAT

Simultaneous flow of heat and electricity in the same direction gives rise to the Thomson effect or the Thomson heat. In the case of a disc, the total generated heat can be expressed as

$$q = \rho(i/2\pi rt)^2 - \sigma(i/2\pi rt)(\mathrm{d}T/\mathrm{d}r). \tag{15}$$

Where the first and second terms on the right-hand side correspond to the Joule and the Thomson effects, respectively. Substituting equation (15) in equation (1) and rearranging, one obtains

$$r^2 \frac{\mathrm{d}^2 T}{\mathrm{d}r^2} + (1 - N) r \frac{\mathrm{d}T}{\mathrm{d}r} + M = 0 \tag{16}$$

where

$$M \equiv (\rho/k)(i/2\pi t)^2 \tag{7}$$

and

$$N \equiv (\sigma/k)(i/2\pi t). \tag{17}$$

The general solution of equation (16) is

$$T = (M/N) \ln r + C_3 r^N + C_4. \tag{18}$$

The specific solutions of equation (18) are obtained for the two different boundary conditions given by equations (9) and (11).

Case 1

$$T - T_0 = [T_i - T_0 + (M/N) \ln (r_0/r_i)] \times [(r^N - r_0^N)/(r_i^N - r_0^N)] - (M/N) \ln (r_0/r). \tag{19}$$

Case 2

$$T - T_0 = (M/N^2)[(r_0/r)^N - 1] - (M/N)\ln(r_0/r). \qquad (20)$$

The effect of the Thomson heat on the temperature distribution in a disc is negligible for metals and most metallic alloys. However, in semiconductors and certain alloys with high Thomson coefficients the contribution of the Thomson heat may be appreciable.

REFERENCES

1. H. S. CARSLAW and J. C. JAEGER, *Conduction of Heat in Solids*, 2nd edn., Oxford University Press, London (1959).